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Chapter 3

Mathematical Modelling of Batch Distillation Columns: A Comparative Analysis of Non-Linear and Fuzzy Models

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Additional information is available at the end of the chapter

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Abstract

Distillation is the process most commonly used in industry to separate chemical mixtures; its applications range from cosmetic and pharmaceutical to petrochemical industries. The equipment required to perform the distillation process is known as distillation column. Since initial investment and maintenance costs for distillation columns are very high it is necessary to have an appropriate mathematical model that allows improving the comprehension of the column dynamics, especially its thermal behaviour, in order to enhance the control and safety of the process. This chapter presents a general panorama of the mathematical modelling of distillation columns, having as a specific case of study the comparison of a space-state non-linear model and a Takagi-Sugeno fuzzy model for a batch distillation column using a binary mixture (Ethanol-Water).

Keywords: mathematical modelling, distillation column, Takagi-Sugeno, non-linear models

1. Introduction

Distillation is the process most commonly used in industry to separate chemical mixtures, being the petrochemical industry one of the most important due to that oil distillation allows obtaining useful product, such as fuels. Distillation is also widely used in the pharmaceutical and cosmetics industry in order to obtain specific drugs and in the liquor industry to obtain wines and liquors, among other applications [1].

Distillation columns are the essential equipment required to perform the distillation process, these columns allow producing food, fuel, medicine, among other products. However, distillation columns represent an important investment in the process they are used, that is why it is necessary to have both, corrective and predictive maintenance, in order to prevent failures in the process as well as in the equipment.

Through the computational and technological continuous development, the industrial processes, such as distillation, have become very complex systems due to the high number of components they have and the several functions they develop, so their vulnerability has also increased. Having appropriate techniques to model distillation columns, such that these models allow implementing efficient and reliable control techniques, is very important to obtain the desired product quality, the adequate process functioning and to improve the security of the system and the user.

In the literature, different mathematical models have been used to improve distillation columns dynamics and comprehensions have been reported. Simple linear and non-linear models are representations that consider only few variables and low-order equations, simplifying the design and implementation of controllers using computational tools. Kienle [2] presents a low-order model for an ideal multicomponent distillation process considering the non-linear wave propagation theory.

Balasubramhanya and Doyle Iii [3] present a low-order model for a reactive multicomponent distillation column as well as the designing of a (MPC) predictive control to obtain the best quality of the distilled product. In Ref. [4], a model based on neural networks having the aim of optimizing the energy efficiency in a binary distillation column is presented. Lopez-Saucedo et al. [5] present the simulation and optimization of a model for a conventional and nonconventional batch distillation column.

Astorga et al. [6] and Cervantes et al. [7] present high-gain observers to estimate the light component composition in a continuous distillation column using a set of models for each plate of the column. In Ref. [8], a fault tolerant scheme for a distillation column, where observers are used to detect failures in the temperature sensors considering a non-linear model of the distillation column, is presented. The parametric identification is other methodology used to estimate certain variables in distillation columns as presented in Refs. [9, 10].

The Takagi-Sugeno fuzzy model is a useful tool to model and control complex systems based on the concept of decomposing a non-linear model in a multi-model structure formed by linear models not necessarily independents and fuzzy logic [11, 12], where the non-linear system representation is obtained through a weighted sum of all the sub-systems. The Takagi-Sugeno fuzzy model provides a solution to solve the designing and implementation issues in control strategies for non-linear systems, for instance, Wang et al. [13] propose a methodology to design control techniques for systems having a Takagi-Sugeno form.

The stability analysis of the Takagi-Sugeno fuzzy model can be solved considering the Lyapunov approach and by using the inner point tool as well as optimization techniques based on linear matrix inequalities (LMIs) [14].

In this chapter, the design and simulation of a non-linear state-space and Takagi-Sugeno models for a batch distillation column are presented. These models are simulated and compared in order to analyse if they aim the objective of representing adequately the process dynamics in order to facilitate the implementation of control strategies to improve the distilled product quality as well as the process security.

2. Distillation column operation modes

Due to the variety of substances found in the nature and their different phases (mainly liquid and vapour), there exist different distillation operation modes in order to separate diverse mixtures, obtaining different quality of products.

The main distillation operation types are as follows:

- *Vacuum distillation:* A low-pressure system is used in order to obtain a low-temperature boiling of the substances in the mixture. Usually, a vacuum pump is used to generate the low-pressure state, as shown in **Figure 1a**.
- *Destructive distillation:* The substance is heated at high temperatures to be decomposed in other products that can be separated by fractionating, its operation is similar to the one used in wood and coal, as shown **Figure 1b**.
- *Extractive distillation:* Different separation agents are added to azeotropic mixtures, altering the relative volatility of the mixture components in order to allow their separation (see **Figure 1c**).
- *Fractionating distillation*: Liquid mixtures are separated by heating, considering a high heat exchange and the liquid and vapour molar rates. This distillation is used to separate composite mixtures/substances having different but close boiling temperatures. It usually considers a continuous operation, having a constant feeding flow through a feeding tray. The section above the feeding tray is named rectifying section, under the feeding tray is called stripping section, as shown in **Figure 2a**.
- *Batch distillation*: Widely used in industry when having small liquid quantities or when obtaining different products from a single mixture load is required. This operation does not have steady state due that the mixture composition varies in time; besides, it only allows enriching or rectifying the distilled (lighter) product (see **Figure 2b**).

In general, the different distillation operation modes have the same operating principle, mainly due the physical variables that interact in the process, such as temperature, composition, pressure and heating energy.

A typical distillation column is formed by a boiler, a condenser and n trays. The boiler is the element that provides the heating energy necessary to evaporate the mixture into it. The condenser provides the cooling necessary to condensate vapour, part of this vapour returns to the column to enrich the mixture, the rest is obtained as a distilled product. The column body is composed of a set of trays, where a partial separation of the mixture is performed due the circulation of liquid and vapour flow.

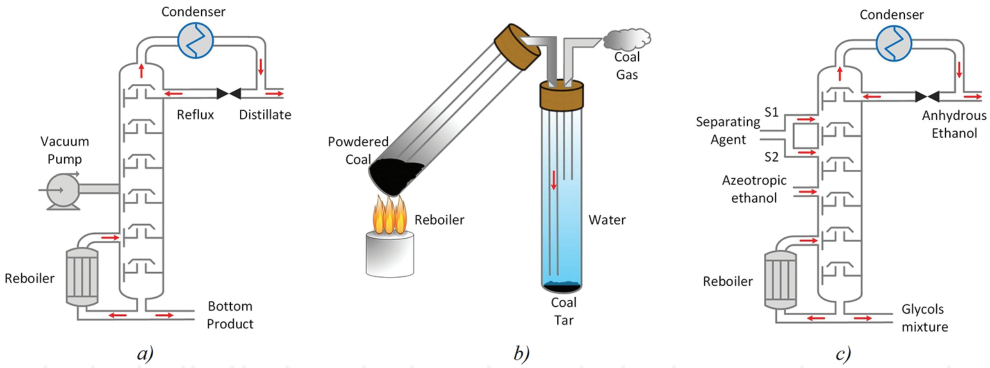

Figure 1. Distillation columns operation modes: (a) vacuum, (b) destructive and (c) extractive.

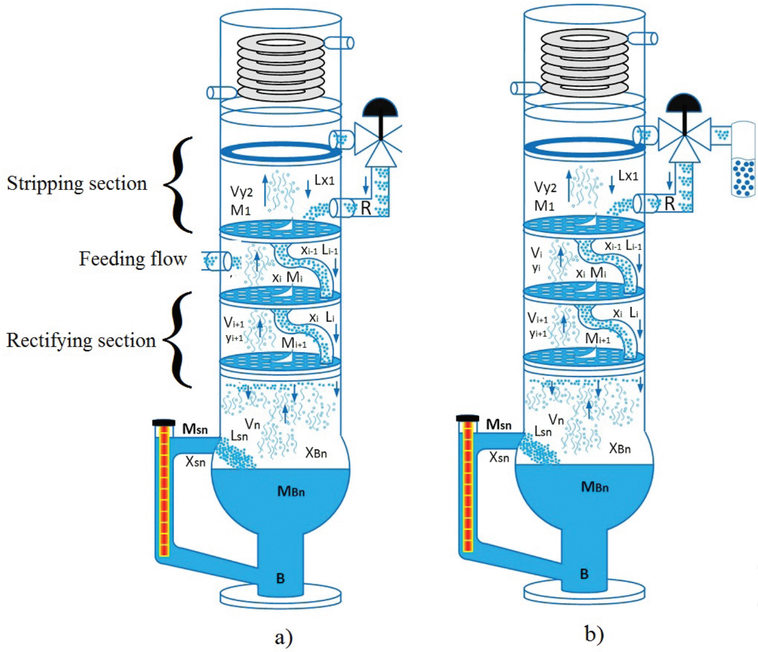

Figure 2. Distillation column operation modes: (a) fractionating and (b) batch.

The vapour flow is generated by the ebullition of the mixture in the boiler, the vapour rises into the column body and it is enriched by the light element of the mixture in each tray of the column. The liquid flow, generated by the reflux, descends from the condenser to the boiler by gravity and it is enriched by the heavy element of the mixture in every tray of the column. This operation can be described by an adequate mathematical model of the process.

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3. Distillation column mathematical modelling

The main objectives of designing a mathematical model of the distillation process are to simplify the analysis and comprehension of the distillation dynamics, facilitate the design of control techniques to enhance the distilled product quality and the system performance, estimate variables difficult to be measured, diagnose failures, among others. In order to deal with these objectives development of an adequate model is indispensable.

There exist different distillation column models according to its operation, the most commonly used in industrial applications are the continuous (fractionating) and batch models. Because of the similarity between the continuous and the batch operating types, in this section, a generic model that presents adequate results in both cases is presented.

It is well known that having a more complete/complex model implies having more complex equations difficult to solve, whereas having a simpler representation implies having simpler equations but the response resolution will have a higher calculation error compared to the real system response.

In general, there are two main model types according their complexity: simple and complex. The simple model found in the literature is the differential model, which considers the boiler and the condenser as trays in the distillation column. The column dynamics is represented by the component mass balance as shown in Eq. (1).

$$\frac{dWx_w}{dt} = \frac{Wdx_w}{dt} + \frac{x_w dW}{dt} = -Dy_D \tag{1}$$

where W is the bottom product, x_W is the bottom product composition, D is the distilled product and y_D is the distilled product composition.

The complex model considers each column element individually, i.e. a condenser, a boiler and trays are modelled individually, such that the response has a better resolution.

The particular study case presented in this chapter considers a complex model of a batch distillation column using a binary mixture.

4. Non-linear model of a binary batch distillation column

The model for a binary batch distillation column is obtained considering the light component, this component is obtained as a final (distilled) product [15]. The light component composition is obtained in each tray of the distillation column, where the liquid and vapour molar flows interact.

In order to design the distillation column model, the following assumptions are considered [16]: total condenser, no heating losses in the body column, constant pressure in the body column, liquid and vapour phases in thermodynamic equilibrium in each plate, variable relativity volatility according to the component composition.

The distillation column dynamics is represented by a set of differential equations that describe the behaviour of the light component of the mixture, given by Eq. (2).

$$\frac{dx_i}{dt} = \frac{L(x_{i-1}-x_i) + V(y_{i+1}-y_i)}{M_i} \tag{2}$$

where x_i is the liquid molar composition of the light component in tray i, y_i is the vapour molar composition of the light component in tray i, L is the liquid molar flow, V is the vapour molar flow and M is the retained mass.

The phase equilibrium is determined by constant K, as shown in Eq. (3) for ideal mixtures.

$$K = \frac{x_i}{y_i} \tag{3}$$

Such that considering the vapour-liquid equilibrium (VLE) and the relative volatility, the vapour composition as a function of the liquid composition is obtained. This function is presented in Eq. (4).

$$y = f(x, \alpha) \tag{4}$$

This is specifically presented in Eq. (5).

$$y_i = \frac{\alpha_i x_i}{1 + (\alpha_i - 1)x_i} \tag{5}$$

where α is the relative volatility in tray i.

Within each element of the distillation column flow different molar rates/quantities, named molar flows. These flows are the liquid and the vapour entering and leaving each tray, the distilled product and the bottom product.

In a binary batch distillation column, the liquid flows in both rectifying and stripping sections are the same, as well as the vapour flows, because there is not feeding flow.

$$\begin{aligned} L &= L_S = L_R \\ V &= V_S = V_R \end{aligned} \tag{6}$$

The molar flows considered in the binary batch distillation model are four: vapour (V), liquid (L), distilled (D) and bottom (B) products, these are expressed in Eqs. (7)–(9) [17].

$$V = \frac{Q_B}{H_1^{\text{vap}} x_n + H_1^{\text{vap}}(1 - x_n)} \tag{7}$$

where Q_B is the heating power (input), x_n is the liquid composition of light component in the boiler (tray n), H_1^{vap} is the vaporization enthalpy of the light component and H_2^{vap} is the vaporization enthalpy of the heavy component.

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$$L = (1-R)V \tag{8}$$

where R is the reflux input.

$$D = V-L \tag{9}$$

B, the bottom product, is not calculated, it is considered as the molar flow that remains into the boiler.

The non-linear model of the binary batch distillation column presented in this chapter is based on a set of sub-models, each sub-model corresponding to a specific element of the column (boiler, condenser and trays).

4.1. Condenser sub-model

The condenser is numbered as tray 1. Its dynamics is described by Eq. (10).

$$\frac{dx_1}{dt} = \frac{Vy_2-Lx_1-Dx_1}{M_1} \tag{10}$$

By substituting $L = (1-R)V$ in $D = V-L$, in order to represent the condenser as a function of the reflux, Eq. (11) is obtained.

$$D = RV \tag{11}$$

By substituting Eq. (11) in Eq. (10), Eq. (12) is obtained.

$$\frac{dx_1}{dt} = \frac{Vy_2-Lx_1-RVx_1}{M_1} \tag{12}$$

Considering that

$$y_2 = \frac{\alpha_2 x_2}{1-(\alpha_2-1)x_2} \tag{13}$$

the non-linear equation that represents the condenser dynamics is finally represented in Eq. (14).

$$\frac{dx_1}{dt} = \frac{V}{M_1}\left(\frac{\alpha_2 x_2}{1-(\alpha_2-1)x_2}\right)-\frac{Lx_1}{M_1}-\frac{RVx_1}{M_1} \tag{14}$$

4.2. Tray sub-model

The column body is formed by a set of n-2 trays. Eq. (15) describes its dynamics.

$$\frac{dx_i}{dt} = \frac{Vy_{i+2}-Vy_i + Lx_{i-1}-Lx_i}{M_i}; \quad i = 2, 3, \ldots, n-1 \tag{15}$$

where n is the total number of trays including a boiler and a condenser.

Considering that

$$y_i = \frac{\alpha_i x_i}{1-(\alpha_i-1)x_i} \tag{16}$$

the non-linear equation that represents the condenser dynamics is finally represented in Eq. (17).

$$\frac{dx_i}{dt} = \frac{V}{M_i}\left(\frac{\alpha_{i+1}x_{i+1}}{1-(\alpha_{i+1}-1)x_{i+1}}\right) - \frac{V}{M_i}\left(\frac{\alpha_i x_i}{1-(\alpha_i-1)x_i}\right) + \frac{L(x_{i-1}-x_i)}{M_i} \tag{17}$$

4.3. Boiler sub-model

The boiler is numbered as tray n. Eq. (18) describes its dynamics.

$$\frac{dx_n}{dt} = \frac{Vx_n - Vy_n + Lx_{n-1} - Lx_n}{M_n} \tag{18}$$

Factorizing Eq. (18), Eq. (19) is obtained.

$$\frac{dx_n}{dt} = \frac{V(x_n-y_n) + L(x_{n-1}-x_n)}{M_n} \tag{19}$$

Solving V to represent Eq. (19) as a function of the heating power (Q_B) based on Eq. (7), Eq. (20) is obtained.

$$\frac{dx_n}{dt} = \left(\frac{Q_B}{H_1^{\mathrm{vap}}x_n + H_1^{\mathrm{vap}}(1-x_n)}\right)\left(\frac{x_n-y_n}{M_n}\right) + \frac{L(x_{n-1}-x_n)}{M_n} \tag{20}$$

Then, considering

$$y_n = \frac{\alpha_n x_n}{1-(\alpha_n-1)x_n} \tag{21}$$

the non-linear equation that represents boiler dynamics is finally represented in Eq. (22).

$$\frac{dx_n}{dt} = \left[\left(\frac{Q_B}{H_1^{\mathrm{vap}}x_n + H_1^{\mathrm{vap}}(1-x_n)}\right)\left(\frac{x_n}{M_n}\right)\left(\frac{1-\alpha_n}{1-(\alpha_n-1)x_n}\right)\right] + \frac{L(x_{n-1}-x_n)}{M_n} \tag{22}$$

4.4. State-space non-linear model for a binary batch distillation column

In this section, the distillation column sub-models shown in Eqs. (14), (17) and (22) are presented in a state-space representation having the form shown in Eq. (23).

$$\dot{x} = Ax + Bu \tag{23}$$

This representation is used in a specific study case, a 12-tray distillation column including a boiler and a condenser, using a binary mixture in a batch operation. Compositions

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$x = [x_1, x_2, \ldots, x_{12}]$ are considered as states of the model and $u = [R, Q_B]^T$ as inputs of the model. Matrices A and B are shown in Eqs. (24) and (25), respectively.

$$A = \begin{bmatrix} -\frac{L}{M_1} & \frac{Vf(x_2,\alpha_2)}{M_1} & 0 & 0 & \cdots & 0 & 0 \\ \frac{L}{M_2} & \frac{-L-Vf(x_2,\alpha_2)}{M_2} & \frac{Vf(x_3,\alpha_3)}{M_2} & 0 & \cdots & 0 & 0 \\ 0 & \frac{L}{M_3} & \frac{-L-Vf(x_3,\alpha_3)}{M_3} & \frac{Vf(x_4,\alpha_4)}{M_3} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{L}{M_{12}} & -\frac{L}{M_{12}} \end{bmatrix} \tag{24}$$

$$B = \begin{bmatrix} \frac{Vx_1}{M_1} & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & \frac{x_{12}-f(x_{12}-\alpha_{12})}{\left(H_1^{vap}x_{12} + H_1^{vap}(1-x_{12})\right)M_{12}} \end{bmatrix} \tag{25}$$

5. Takagi-Sugeno fuzzy model for a binary batch distillation column

The Takagi-Sugeno fuzzy representation describes the system dynamics based on linear sub-models interpolation and fuzzy rules [18].

Rule for model *j:*

If $z_1(t)$ is M_{1jj}, $z_2(t)$ is M_{2j},… and $z_p(t)$ is M_{pj}

Then:

$$x(t) = \sum_{j=1}^{r} A_j x(t) + B_j u(t) \tag{26}$$

where $j = 1, 2, \ldots, r$, M_j is the fuzzy set, r is the sub-model number, x is the state vector, u is the input vector, A_j is the state matrix for sub-model j, B_j is the input matrix for sub-model j and $z_j(t)$ is the scheduling measurable variable (state variables or external disturbances).

Given $[x(t,), u(t), z(t)]$, the complete fuzzy model is obtained by using a singleton-type fuzzifier, a product-type defuzzifier mechanism and the gravity centre. The Takagi-Sugeno fuzzy model for the non-linear system is expressed in Eq. (27).

$$\dot{x}(t) = \frac{\sum_{j=1}^{r}\omega_j\left(z_j(t)\right)[A_jx(t) + B_ju(t)]}{\sum_{j=1}^{r}\omega_j\left(z_j(t)\right)} \tag{27}$$

where the weight $\omega_j\left(z_j(t)\right)$ is 0 or a positive value, such that the sum of all the weights is positive; thus, the normalized weight, *hi*, is calculated in every rule from the z_j membership functions in the M_{jk} set. It is well known by fuzzy logic that $h_j = h_j[z(t)] \geq 0$ and $\sum_{j=1}^{r}h_j[z_j(t)] = 1$, as expressed in Eq. (28).

$$h_j[z_j(t)] = \frac{\omega_j\left(z_j(t)\right)}{\sum_{j=1}^{r}\omega_j\left(z_j(t)\right)} \tag{28}$$

The system expressed in Eq. (27) is equivalent to the system in Eq. (29).

$$\dot{x}(t) = \sum_{j=1}^{r}h_j[A_jx(t) + B_ju(t)] \tag{29}$$

5.1. Application to a binary batch distillation column

In this chapter, the specific study case is a 12-tray distillation column, including a boiler and a condenser, using an ethanol-water mixture in a batch operation. In the Takagi-Sugeno fuzzy model the liquid (L) and vapour (V) molar flows are proposed as parameters; the nominal operating ranges in steady state are:

$$\begin{aligned} L &= [0.418783, 2.97801] \\ V &= [0.418783, 2.97801] \end{aligned} \tag{30}$$

According to these parameters, the Takagi-Sugeno fuzzy model that interpolates between four linear models based on the following rules is obtained:

Rule 1:

$$\text{if } V \text{ is } V_{\min} \text{ and if } L \text{ is } L_{\min} \tag{31}$$

Then:

$$\dot{x}_1(t) = A_1x(t) + B_1u(t) \tag{32}$$

Rule 2:

$$\text{if } V \text{ is } V_{\min} \text{ and if } L \text{ is } L_{\max} \tag{33}$$

Then:

$$\dot{x}_2(t) = A_2x(t) + B_1u(t) \tag{34}$$

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Rule 3:

$$\text{if } V \text{ is } V_{\max} \text{ and if } L \text{ is } L_{\min} \tag{35}$$

Then:

$$\dot{x}_3(t) = A_3 x(t) + B_2 u(t) \tag{36}$$

Rule 4:

$$\text{if } V \text{ is} V_{\max} \text{ and if } L \text{ is } L_{\max} \tag{37}$$

Then:

$$\dot{x}_4(t) = A_4 x(t) + B_2 u(t) \tag{38}$$

where:

$$\begin{aligned} A_1 &= \{V_{\min}, L_{\min}, G\{x_1, \alpha_1\}, \ldots, G\{x_{12}, \alpha_{12}\}, M_1, \ldots, M_{12}\} \\ A_2 &= \{V_{\min}, L_{\max}, G\{x_1, \alpha_1\}, \ldots, G\{x_{12}, \alpha_{12}\}, M_1, \ldots, M_{12}\} \\ A_3 &= \{V_{\max}, L_{\min}, G\{x_1, \alpha_1\}, \ldots, G\{x_{12}, \alpha_{12}\}, M_1, \ldots, M_{12}\} \\ A_4 &= \{V_{\max}, L_{\max}, G\{x_1, \alpha_1\}, \ldots, G\{x_{12}, \alpha_{12}\}, M_1, \ldots, M_{12}\} \end{aligned} \tag{39}$$

$$\begin{aligned} B_1 &= \begin{pmatrix} \frac{V_{\min} \cdot x_1}{M_1} & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & \frac{x_{12} - g(x_{12}, \alpha_{12})}{(H_{\text{EtoH}}^{\text{vap}} x_{12} + H_{\text{H}_2\text{O}}^{\text{vap}} (1 - x_{12})) \cdot M_{12}} \end{pmatrix} \\ B_2 &= \begin{pmatrix} \frac{V_{\max} \cdot x_1}{M_1} & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & \frac{x_{12} - g(x_{12}, \alpha_{12})}{(H_{\text{EtoH}}^{\text{vap}} x_{12} + H_{\text{H}_2\text{O}}^{\text{vap}} (1 - x_{12})) \cdot M_{12}} \end{pmatrix} \end{aligned} \tag{40}$$

The membership functions *($\mu(z)$)* for the fuzzy set are determined by:

Eq. (41) for vapour V:

$$\mu(V) \begin{cases} \mu V_{\min} = \dfrac{V_{\max} - V}{V_{\max} - V_{\min}} \\ \mu V_{\max} = 1 - \mu V_{\min} \end{cases} \tag{41}$$

Eq. (42) for liquid L:

$$\mu(V)\begin{cases}\mu L_{\min} = \dfrac{L_{\max}-L}{L_{\max}-L_{\min}} \\ \mu L_{\max} = 1-\mu L_{\min}\end{cases} \tag{42}$$

The normalized weights are given by Eq. (43):

$$\begin{cases} h_1(V,L) = \mu V_{\min}\mu L_{\min} \\ h_2(V,L) = \mu V_{\min}\mu L_{\max} \\ h_3(V,L) = \mu V_{\max}\mu L_{\min} \\ h_4(V,L) = \mu V_{\max}\mu L_{\max} \end{cases} \tag{43}$$

The Takagi-Sugeno fuzzy model proposed for the distillation column is represented in Eq. (44).

$$x(t) = \sum_{i=1}^{r} h_i(L,V)(A_i x(t) + B_i u(t)) \tag{44}$$

6. Models experimental validation and comparison

The Takagi-Sugeno fuzzy model is validated in Matlab by using experimental data from a 12-tray batch distillation column with variable reflux, using an ethanol-water mixture and considering the characteristics presented in **Table 1**.

Parameter	Value	Units
EtOH volume in boiler	2000	mL
H_20 volume in boiler	2000	mL
Process total pressure	637.42	mmHg

Table 1. Mixture initial parameters.

The initial molar composition of ethanol in the boiler is 0.2216, considering that the feed volume corresponds to 96%Vol ethanol.

The characteristics of the process inputs for the study case, the heating power (Q_B) and the reflux valve opening (R) are shown in **Table 2**.

Figure 3 presents the temperatures estimated by the Takagi-Sugeno model for the trays in the column body. The temperature increment and decrement due the reflux (R) action can be seen in all the trays.

Figure 4 presents the temperature graphics corresponding to the condenser (a) and to the boiler (b) in the non-linear and Takagi-Sugeno model. Temperature variations existing during the heating power (Q_B) and reflux changes (R) are shown. It can be seen that there exist a difference between the results obtained by both models due the reflux action, this difference is provoked by the fixed operating points for liquid and vapour flows in the Takagi-Sugeno model; however, this difference is small (less than 1.5%).

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Input	Signal	Time
Q_B	Step 0–800 J	0 min
R	Total	0 min
Q_B	Step 800–1000 J	3.3 min
Q_B	Step 1000–1250 J	5.98 min
R	Pulse (ton = 6 s, toff = 6 s)	12.61 min
Q_B	Step 1250–1100 J	14.78 min
Q_B	Step 1100–950 J	17.15 min
Q_B	Step 950–1100 J	19.36 min
R	Total	23 min
Q_B	Step1100–1250 J	24.88 min

Table 2. Input parameters.

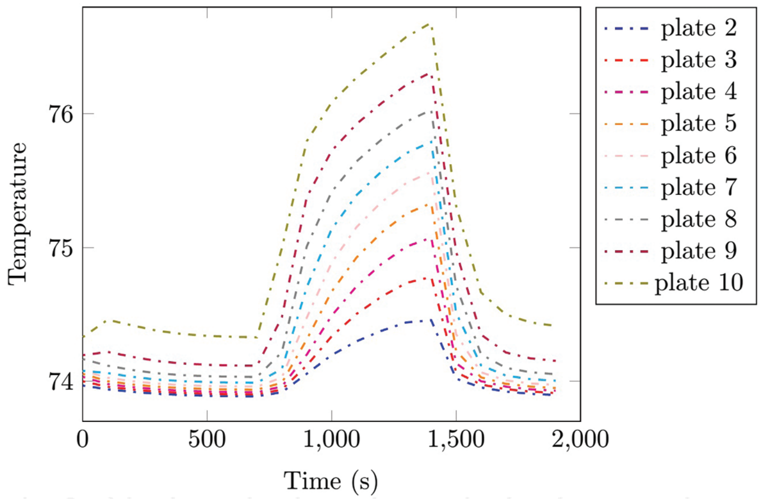

Figure 3. Plate temperatures in the distillation column.

In **Figure 5**, the composition graphics estimated for the distillation column trays by the Takagi-Sugeno fuzzy model are presented, these composition values vary according to the tray position.

In **Figure 6**, the simulation results obtained by the non-linear and Takagi-Sugeno models for the light component composition in the condenser (a) and the boiler (b) are presented. It can be seen that the composition behaviour in both trays varies according the heating power (Q_B) and reflux (R) changes, as shown in **Table 2**.

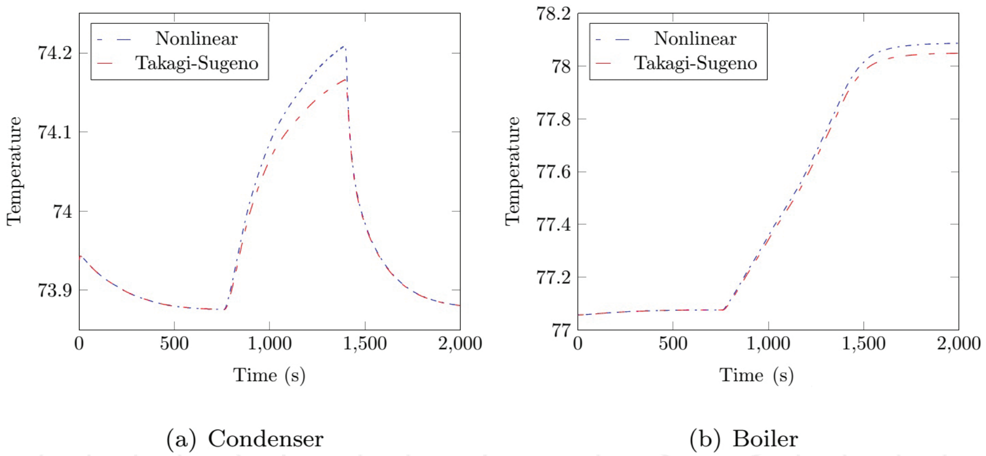

Figure 4. a) Condenser and b) Boiler temperatures, non-linear and Takagi-Sugeno models.

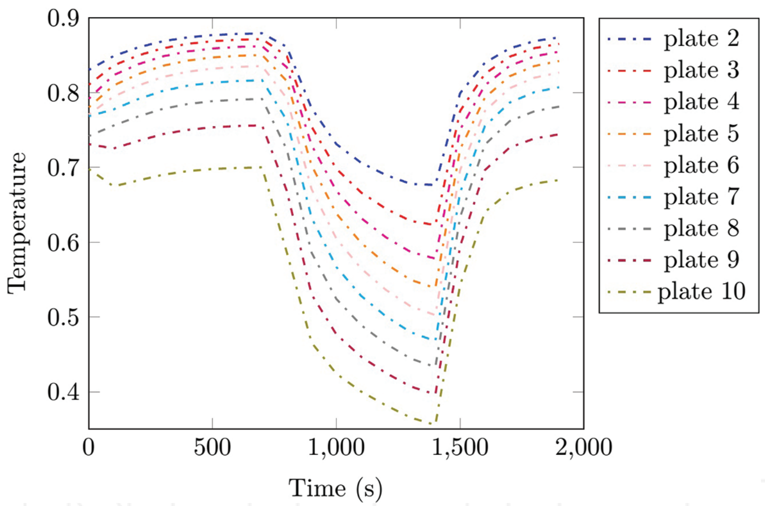

Figure 5. Plate temperatures in the distillation column.

Figure 7 shows the liquid and vapour molar flow behaviour during the distillation process. It can be seen the process dynamics when reflux or heating power changes exist.

The error percentage in the Takagi-Sugeno model compared to the non-linear models, calculated by the function shown in Eq. (45), is graphically represented in **Figure 8**. It can be seen that the error behaviour in the condenser (a) and the boiler (b) has a maximum value of 1.5% due to the reflux changes.

$$e = \frac{|x_{TS} - x_{NL}|}{x_{NL}} 100\% \tag{45}$$

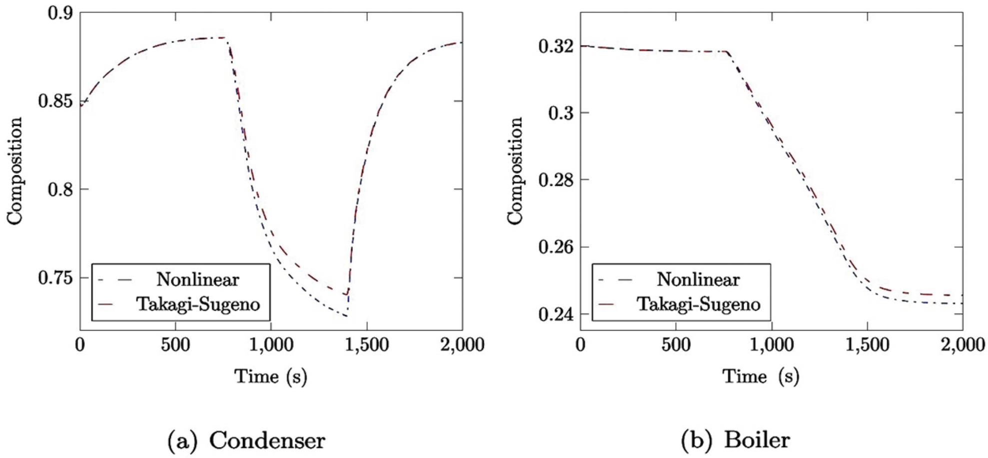

Figure 6. a) Condenser and b) Boiler temperatures obtained by non-linear and Takagi-Sugeno models.

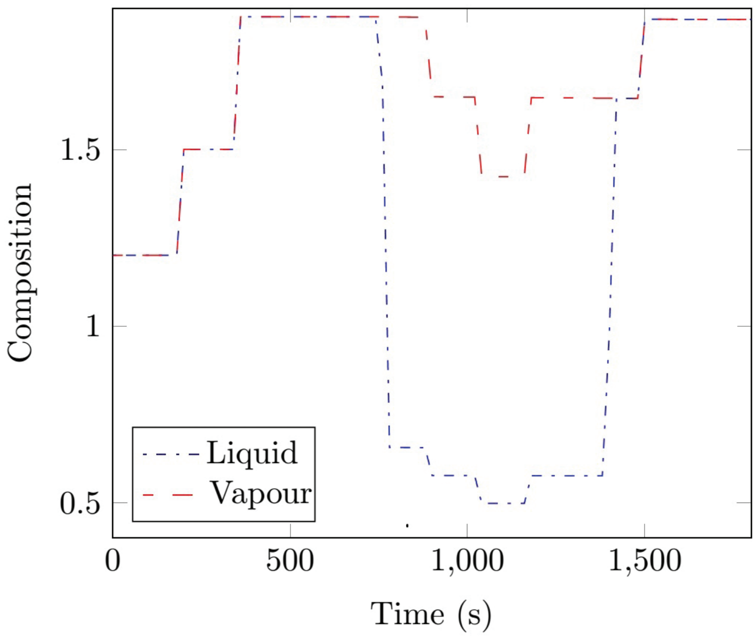

Figure 7. Liquid and vapour molar flows.

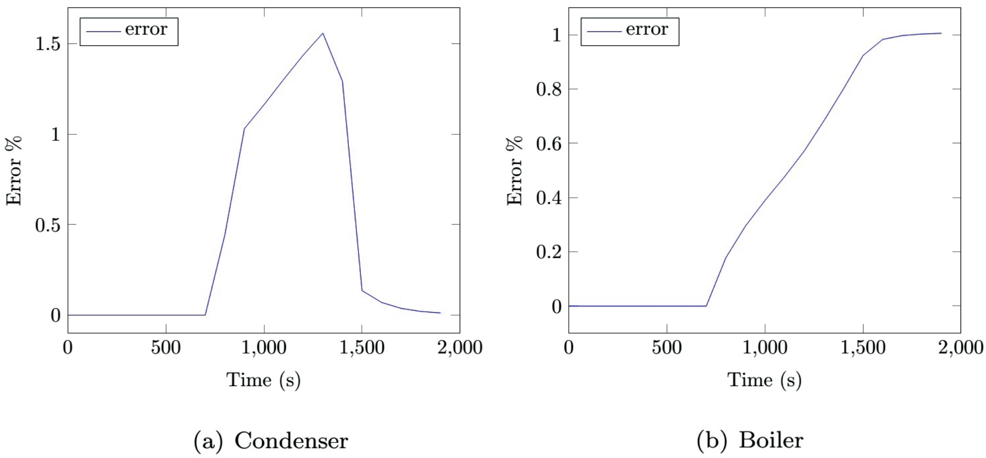

Figure 8. a) Condenser and b) Boiler error percentages.

7. Conclusions

This chapter presents the analysis and design of a state-space non-linear model and the Takagi-Sugeno fuzzy model for a batch distillation column using a binary mixture. The state-space non-linear model is based on differential equations considering compositions, temperatures and molar flows in the column. The linear fuzzy model is based on four rules, considering as parameters the liquid and vapour molar flows.

Both, the state-space non-linear and the linear fuzzy models are simulated in Matlab considering real input parameters (heating power and reflux) from a 12-tray batch distillation pilot plant using an ethanol-water mixture. The light component compositions and the temperatures in each tray of the column are calculated by both models. Besides, the obtained results are compared considering the same operating parameters, this comparison has the aim to verify the adequate functioning of the non-linear state-space and the Takagi-Sugeno models in order to analyse the existing differences.

The Takagi-Sugeno fuzzy model presents small differences in the estimations of the composition component and the tray temperatures when a reflux disturbance is presented due that the reflux affects directly the operating points established in this model; however, these differences are small enough to be neglected and both models converge under any operating condition.

The Takagi-Sugeno fuzzy model for a distillation column represents an alternative tool that takes advantage of the fuzzy control theory, allowing to facilitate the design and implement nonconventional control strategies for non-linear systems, however, if a higher resolution response is required it could be convenient to consider the non-linear model.

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