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# Non-Orthogonal Multiple Access (NOMA) for 5G Networks

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## Abstract

In this chapter, we explore the concept of non-orthogonal multiple access (NOMA) scheme for the future radio access for 5G. We first provide the fundamentals of the technique for both downlink and uplink channels and then discuss optimizing the network capacity under fairness constraints. We further discuss the impacts of imperfect receivers on the performance of NOMA networks. Finally, we discuss the spectral efficiency (SE) of the networks that employ NOMA with its relations with energy efficiency (EE). We demonstrate that the networks with NOMA outperform other multiple access schemes in terms of sum capacity, EE and SE.

**Keywords:** non-orthogonal multiple access (NOMA), energy efficiency, power efficiency

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## 1. Introduction

In this chapter, we explore the concept of non-orthogonal multiple access (NOMA) method for the upcoming 5G networks. All of the current cellular networks implement orthogonal multiple access (OMA) techniques such as time division multiple access (TDMA), frequency division multiple access (FDMA) or code division multiple access (CDMA) together. However, none of these techniques can meet the high demands of future radio access systems.

The characteristics of the OMA schemes can be summarized as follows. In TDMA, the information for each user is sent in non-overlapping time slots [1], so that TDMA-based networks require accurate timing synchronization, which can be challenging, particularly in

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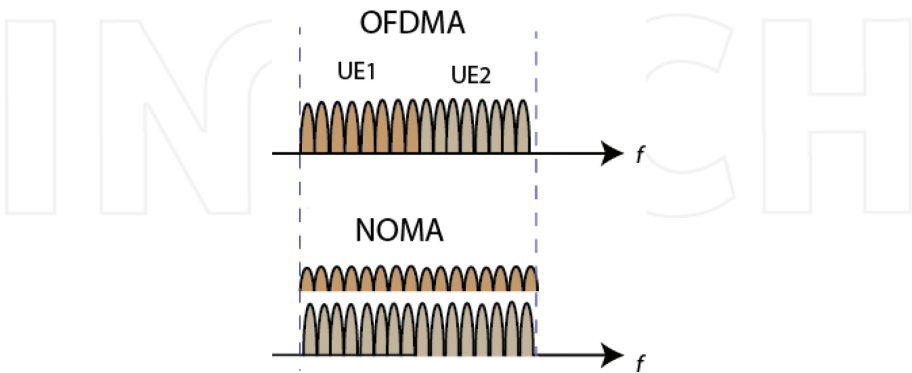
the uplink. In FDMA implementations, such as orthogonal frequency division multiple access (OFDMA), information for each user is assigned to a subset of subcarriers [1]. CDMA utilizes codes in order to separate the users over the same channel [1]. NOMA is fundamentally different than these multiple access schemes which provide orthogonal access to the users either in time, frequency, code or space. In NOMA, each user operates in the same band and at the same time where they are distinguished by their power levels. NOMA uses superposition coding at the transmitter such that the successive interference cancellation (SIC) receiver can separate the users both in the uplink and in the downlink channels.

NOMA was proposed as a candidate radio access technology for 5G cellular systems [2, 3]. Practical implementation of NOMA in cellular networks requires high computational power to implement real-time power allocation and successive interference cancellation algorithms. By 2020, the time that 5G networks are targeted to be deployed, the computational capacity of both handsets and access points is expected to high enough to run NOMA algorithms.

In this chapter, we present the fundamentals and capacity limits of NOMA as a future radio access technology. The imperfectness in the SIC receiver and its impact on the overall capacity is also presented. We further contribute to the literature by demonstrating the improved energy and spectral efficiencies with NOMA over-conventional OFDMA.

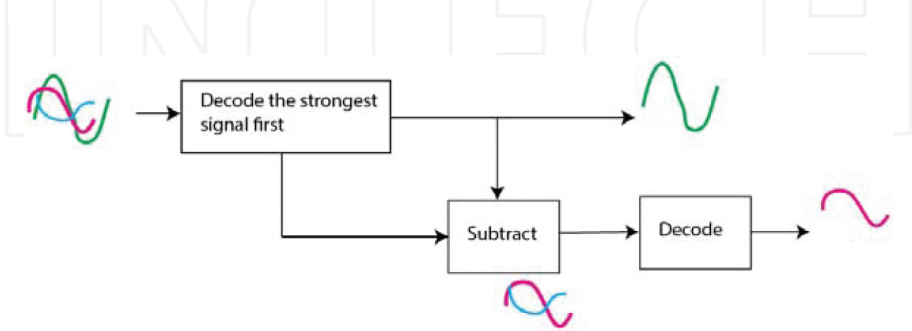
## 2. Non-orthogonal multiple access (NOMA)

We consider orthogonal frequency division multiplexing (OFDM) as the modulation scheme and NOMA as the multiple access scheme. In conventional 4G networks, as natural extension of OFDM, orthogonal frequency division multiple access (OFDMA) is used where information for each user is assigned to a subset of subcarriers. In NOMA, on the other hand, all of the subcarriers can be used by each user. **Figure 1** illustrates the spectrum sharing for OFDMA and NOMA for two users. The concept applies both uplink and downlink transmission.



**Figure 1.** Spectrum sharing for OFDMA and NOMA for two users.

Superposition coding at the transmitter and successive interference cancellation (SIC) at the receiver makes it possible to utilize the same spectrum for all users. At the transmitter site, all the individual information signals are superimposed into a single waveform, while at the receiver, SIC decodes the signals one by one until it finds the desired signal. **Figure 2** illustrates the concept. In the illustration, the three information signals indicated with different colors are superimposed at the transmitter. The received signal at the SIC receiver includes all these three signals. The first signal that SIC decodes is the strongest one while others as interference. The first decoded signal is then subtracted from the received signal and if the decoding is perfect, the waveform with the rest of the signals is accurately obtained. SIC iterates the process until it finds the desired signal.



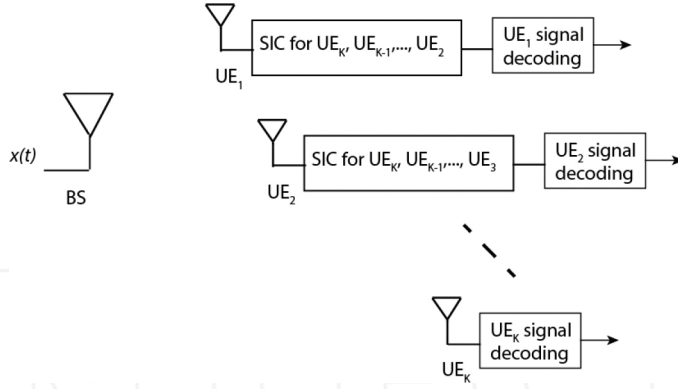
**Figure 2.** Successive interference cancellation.

The success of SIC depends on the perfect cancellation of the signals in the iteration steps. The transmitter should accurately split the power between the user information waveforms and superimpose them. The methodology for power split differs for uplink and downlink channels.

## 2.1. NOMA for downlink

In NOMA downlink, the base station superimposes the information waveforms for its serviced users. Each user equipment (UE) employs SIC to detect their own signals. **Figure 3** shows a BS and  $K$  number of UEs with SIC receivers. In the network, it is assumed that the  $UE_1$  is the closest to the base station (BS), and  $UE_K$  is the farthest.

The challenge for BS is to decide how to allocate the power among the individual information waveforms, which is critical for SIC. In NOMA downlink, more power is allocated to UE located farther from the BS and the least power to the UE closest to the BS. In the network, all UEs receive the same signal that contains the information for all users. Each UE decodes the strongest signal first, and then subtracts the decoded signal from the received signal. SIC receiver iterates the subtraction until it finds its own signal. UE located close to the BS can cancel the signals of the farther UEs. Since the signal of the farthest UE contributes the most to the received signal, it will decode its own signal first.



**Figure 3.** Downlink NOMA for K users.

The transmitted signal by the BS can be written as

$$x(t) = \sum_{k=1}^K \sqrt{\alpha_k P_T} x_k(t) \quad (1)$$

where  $x_k(t)$  is the individual information conveying OFDM waveform,  $\alpha_k$  is the power allocation coefficient for the  $UE_k$ , and  $P_T$  is the total available power at the BS. The power allocated to each  $UE_k$  then becomes  $P_k = \alpha_k P_T$ . The power is allocated according to the distance of UEs to the BS:  $UE_1$  is the closest to the BS, so it is allocated the least power, whereas  $UE_K$  is the farthest one, therefore it has the highest power.

The received signal at the  $UE_k$  is

$$y_k(t) = x(t)g_k + w_k(t) \quad (2)$$

where  $g_k$  is the channel attenuation factor for the link between the BS and the  $UE_k$ , and  $w_k(t)$  is the additive white Gaussian noise at the  $UE_k$  with mean zero and density  $N_0$  (W/Hz).

Let us consider the farthest user first. The signal it decodes first will be its own signal since it is allocated the most power as compared the others. The signals for other users will be seen as interference. Therefore, the signal-to-noise ratio (SNR) for  $UE_K$  can be written as [1]

$$SNR_K = \frac{P_K g_K^2}{N_0 W + \sum_{i=1}^{K-1} P_i g_K^2} \quad (3)$$

where  $W$  is the transmission bandwidth.

For the closest UE<sub>1</sub>, the last signal it decodes will be its signal. Assuming perfect cancellation, the SNR for UE<sub>1</sub> becomes

$$SNR_1 = \frac{P_1 g_1^2}{N_0 W}. \quad (4)$$

In general, for the UE<sub>k</sub>, the SNR becomes

$$SNR_k = \frac{P_k g_k^2}{N_0 W + \sum_{i=1}^{k-1} P_i g_k^2}. \quad (5)$$

When NOMA is used, the throughput (bps) for each UE can be written as

$$R_k = W \log_2 \left( 1 + \frac{P_k g_k^2}{N + \sum_{i=1}^{k-1} P_i g_k^2} \right). \quad (6)$$

In OFDMA, on the other hand, UEs are assigned to a group of subcarriers in order to receive their information. When the total bandwidth and power are shared among the UEs equally, the throughput for each UE for OFDMA becomes

$$R_k = W_k \log_2 \left( 1 + \frac{P_k g_k^2}{N_k} \right) \quad (7)$$

where  $W_k = \frac{W}{K}$  and  $N_k = N_0 W_k$ .

The sum capacity for both OFDMA and NOMA can be written as

$$R_T = \sum_{k=1}^K R_k. \quad (8)$$

We further define fairness index as [4]

$$F = \frac{\left( \sum R_k \right)^2}{K \sum R_k^2} \quad (9)$$

which indicates how fair the system capacity is shared among the UEs, that is, when  $F$  gets close to 1, the capacity for each UE gets close to each other.

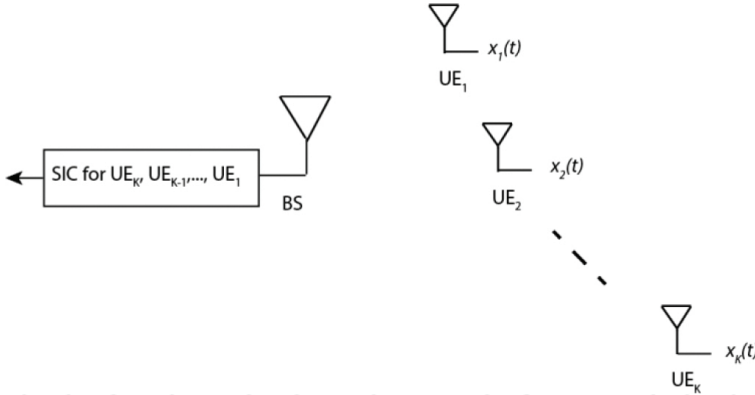
We can set the objective of the power allocation mechanism as to maximize the sum capacity  $R_T$  under a fairness constraint for NOMA systems. The optimization problem is then formulated as

$$\begin{aligned} & \underset{\alpha_k}{\text{maximize}} W \log_2 \left( 1 + \frac{P_k g_k^2}{N + \sum_{i=1}^{k-1} P_i g_k^2} \right) & \text{subject to: } & \sum_{k=1}^K P_k \leq P_T \\ & & & P_k \geq 0, \forall k \\ & & & F = F' \end{aligned} \quad (10)$$

where  $F'$  is the target fairness index in the network. The power allocation coefficients  $\alpha_k$  for each UE <sub>$k$</sub>  can be obtained with exhaustive search.

## 2.2. NOMA for uplink

Uplink implementation of NOMA is slightly different than the downlink. **Figure 4** depicts a network that multiplexes  $K$  UEs in the uplink using NOMA. This time, BS employs SIC in order to distinguish the user signals.



**Figure 4.** Uplink NOMA for  $K$  users.

In the uplink, the received signal by the BS that includes all the user signals is written as

$$y(t) = \sum_{k=1}^K x_k(t) g_k + w(t) \quad (11)$$

where  $g_k$  is the channel attenuation gain for the link between the BS and the UE<sub>k</sub>,  $x_k(t)$  is the information waveform for the  $k$ th UE, and  $w(t)$  is the additive white Gaussian noise at the BS with mean zero and density  $N_0$  (W/Hz). In the uplink, the UEs may again optimize their transmit powers according to their locations as in the downlink. However, here we assume that the users are well distributed in the cell coverage, and the received power levels from different users are already well separated. This assumption is more natural from practical point of view, since power optimization requires connection between all the UEs which may be difficult to implement.

At the receiver, the BS implements SIC. The first signal it decodes will be the signal from the nearest user. The SNR for the signal for the UE<sub>1</sub> can be written as, including others as interference,

$$R_1 = \frac{Pg_1^2}{N + \sum_{i=2}^K Pg_i^2} \quad (12)$$

where  $P$  is the transmission power of UEs and  $N = N_0W$ .

The last signal that the BS decodes is the signal for the farthest user UE<sub>K</sub>. Assuming perfect cancellation, the SNR for UE<sub>K</sub> can be written as

$$SNR_K = \frac{Pg_K^2}{N}. \quad (13)$$

Generally, for the  $k$ th UE, the SNR becomes,

$$SNR_k = 1 + \frac{Pg_k^2}{N + \sum_{i=k+1}^K Pg_i^2}. \quad (14)$$

The throughput (bps) for each UE can be written as

$$R_k = W \log_2 \left( 1 + \frac{Pg_k^2}{N + \sum_{i=k+1}^K Pg_i^2} \right). \quad (15)$$

In OFDMA, on the other hand, UEs are allocated orthogonal carriers in order to receive their information. When the total bandwidth and power are shared among the UEs equally, the throughput for each UE for OFDMA becomes



$$R_k = W_k \log_2 \left( 1 + \frac{P_k g_k^2}{N_k} \right) \quad (16)$$

where  $W_k = \frac{W}{K}$  and  $N_k = N_0 W_k$ .

The sum capacity for both OFDMA and NOMA can be written as

$$R_T = \sum_{k=1}^K R_k. \quad (17)$$

### 3. Imperfectness in NOMA

Our discussions so far in the previous sections assume perfect cancellation in the SIC receiver. In actual SIC, it is quite difficult to subtract the decoded signal from the received signal without any error. In this section, we revisit the NOMA concept with cancellation error in the SIC receiver.

Here, we consider the downlink only; however, the discussions can easily be extended for the uplink. Recall that SIC receiver decodes the information signals one by one iteratively to obtain the desired signal. In SIC, after decoding the signal, one should regenerate the original individual waveform in order to subtract it from the received signal. Although it is theoretically possible to complete this process without any error, in practice, it is expected to experience some cancellation error.

In downlink, the SNR for the  $k$ th user with cancellation error is written as [5]

$$SNR_k = \frac{P_k g_k^2}{N_0 W + \sum_{i=1}^{k-1} P_i g_k^2 + \epsilon \sum_{i=k+1}^K P_i g_k^2}. \quad (18)$$

where  $\epsilon$  is cancellation error term that represents the remaining portion of the cancelled message signal. In the previous section, the third term in the denominator is not included since perfect cancellation is assumed there.

### 4. Spectral efficiency and energy efficiency

Most analysis so far included the throughput performance of the network. In addition to spectral efficiency (SE) of NOMA, in this section, we analyze the energy efficiency (EE) of

NOMA systems. In our analysis, we incorporate the static power consumption of the network due to the power amplifiers in addition to the power consumed for the information waveform.

The total power consumption at the transmitter can be represented as the sum of the information signal power and the power consumed by the circuits (mainly by power amplifiers). Considering the downlink, the total power consumed by the BS can then be written as

$$P_{total} = P_T + P_{static} \quad (19)$$

where  $P_T$  is the total signal power as mentioned earlier and  $P_{static}$  is the power consumed by the circuitry.

Energy efficiency (EE) is defined as the sum rate over the total consumed power of the base-station [6]

$$EE = \frac{R_T}{P_{total}} = SE \frac{W}{P_{total}} \text{ (bits/joule)} \quad (20)$$

where SE is the spectral efficiency ( $R_T/W$ ) in terms of bps/Hz.

The energy efficiency and spectral efficiency relationship (EE-SE) in Shannon theory does not consider the power consumption of the circuit and consequently is monotonic where a higher SE always results in a lower EE. When the circuit power is considered, the EE increases in the low SE region and decreases in the high SE region. The peak of the curve (or the corresponding derivative of the EE-SE relationship) is where the system has the maximum energy efficiency. This point is called “green point” [6–8]. For a fixed  $P_{total}$ , the EE-SE relationship is linear with a positive slope of  $R_T/P_{total}$  where an increase in SE simultaneously results in an increase in EE. As we demonstrate in the next section, NOMA provides higher energy efficiency than OFDMA.

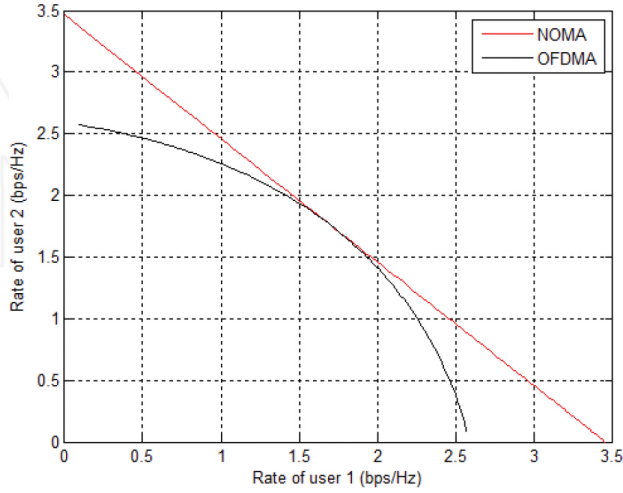
## 5. Results

### 5.1. Rate pairs

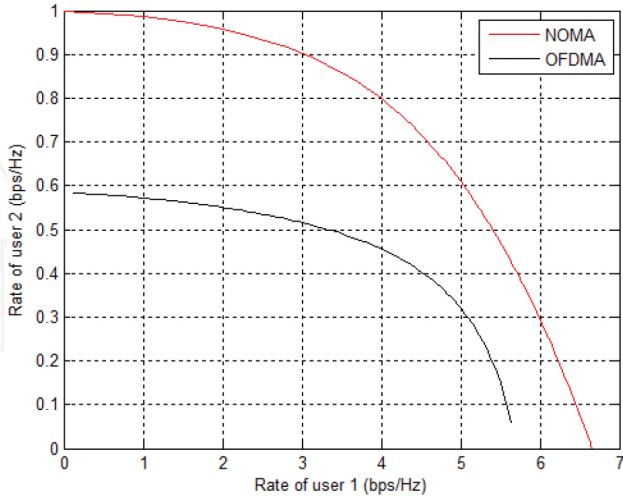
We assume that there are two users in the network for the sake of discussion and analyze the boundaries of the achievable rate regions for these two users. We consider a symmetric downlink channel so that the users are at equal distance to the BS.  $SNR_1 = SNR_2 = 10dB$ .

**Figure 5** shows the boundaries of the achievable rate regions  $R_1$  and  $R_1$  for NOMA and OFDMA. As illustrated in **Figure 5**, NOMA achieves higher rate pairs than the OFDMA except at the corners points (where the rates are equal to the single user capacities). When the fairness is high, both users experience 1.6 bps/Hz throughputs with both NOMA and OFD-

MA. However, when the fairness is lower, both sum capacity and individual throughputs are higher with NOMA. **Figure 6** shows rate pairs when the channel is asymmetric, that is,  $SNR_1 = 20dB$  and  $SNR_2 = 0dB$ . NOMA achieves much higher rate pairs than OFDMA, particularly for the farther user,  $UE_2$ .



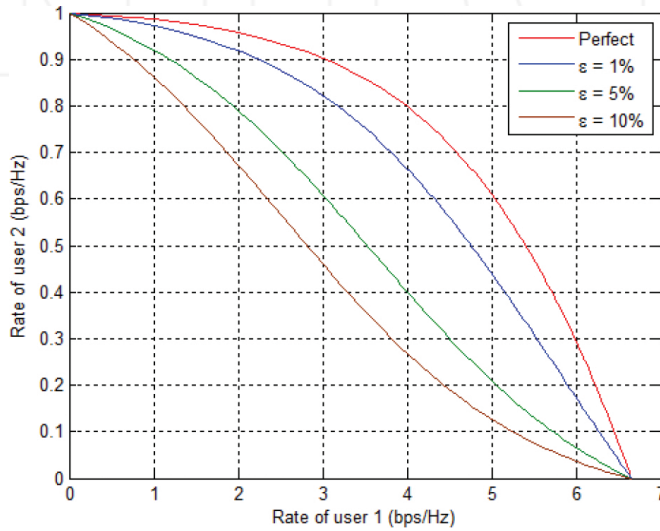
**Figure 5.** Rate pairs with OFDMA and NOMA for downlink NOMA,  $SNR_1 = SNR_2 = 10dB$ .



**Figure 6.** Rate pairs with OFDMA and NOMA for downlink NOMA,  $SNR_1 = 20dB$  and  $SNR_2 = 0dB$ .

## 5.2. Impact of imperfect cancellation

In **Figure 7**, we repeat the same conditions for the asymmetric downlink channel in the previous section with imperfectness in SIC. The case for perfect cancellation is given as reference which is the same as the results in **Figure 6**. We then analyze the impact of imperfect cancellation by setting the cancellation error term ( $\epsilon$ ) at 1, 5 and 10%. For instance, when  $\epsilon = 1\%$ , UE<sub>1</sub> cannot perfectly cancel the signal for UE<sub>2</sub> in the first iteration, and 1% of the power of the second user's signal still remains as interference. When  $\epsilon = 1\%$ , the individual rate pairs and accordingly overall capacity slightly reduce. When  $\epsilon = 10\%$ , on the other hand, the reduction is more distinct.



**Figure 7.** Impact of imperfect cancellation in SIC.

## 5.3. SE-EE trade-off with NOMA

Here, we compare the EE and SE of NOMA with OFDMA. We again consider the downlink. The system bandwidth is taken as  $W = 5$  MHz. The channel gains for UE1 and UE2 are, respectively, taken as  $g_1^2 = -120\text{dB}$  and  $g_2^2 = -140\text{dB}$ . Noise density  $N_0$  is taken as  $-150\text{ dBW/Hz}$ . We assume that the static power consumption at the BS is  $P_{\text{static}} = 100\text{W}$ . **Figure 8** shows the obtained EE-SE curves for this setup. It is seen that NOMA achieves higher EE and SE than OFDMA system. The green-points occur for NOMA and OFDMA when  $P_T$  is at 17 W and 18 W, respectively. At these points, both systems achieve their maximum EE. NOMA clearly outperforms OFDMA at green point and beyond for both EE and SE.

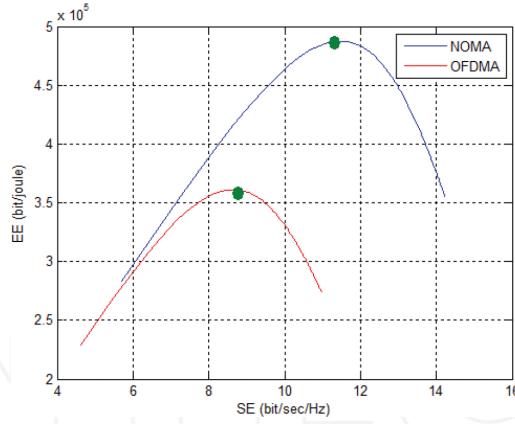


Figure 8. EE-SE trade-off curves for NOMA and OFDMA.

## 6. Conclusion

In this chapter, we have presented the fundamentals of NOMA and demonstrated its superior performance over conventional OFDMA in terms of sum capacity, energy efficiency and spectral efficiency. We have further mentioned the impact of imperfectness at the SIC receiver on the system performance. With its distinct features, NOMA stays as the strongest candidate for the future 5G networks. There are, however, still some challenges for successful implementation of NOMA. First of all, it requires high computational power to run SIC algorithms particularly for high number of users at high data rates. Second, power allocation optimization remains as a challenging problem, particularly when the UEs are moving fast in the network. Finally, SIC receiver is sensitive to cancellation errors which can easily occur in fading channels. It can be implemented with some other diversity techniques like multiple-input-multiple-output (MIMO) or with coding schemes in order to increase the reliability and accordingly reduce the decoding errors. There are recent works that implement MIMO for NOMA [9, 10]; the impact of channel state information (CSI) is studied in [11], capacity maximization problem is discussed in [11], and outage probability expressions are derived in [12]. The current state of the art for NOMA, however, is still far from its potential and requires further investigation.

## Appendix

### MATLAB code for Figure 5.

```
clear all;
clc;
```

```
%%% NOMA parameters

P = 1;
G1 = 10;
G2 = 10;
count = 1;
for alpha = 0:0.01:1 %power splitting factor
P1 = P*alpha;
P2 = P - P1;
R1(count) = log2(1 + P1*G1);
R2(count) = log2(1 + P2*G2/(P1*G2 + 1));
count = count + 1;
end
hold on;
plot (R1,R2,'r');
grid on;
count = 1;
for alpha = 0:0.01:1 %bandwidth splitting factor
P1 = P/2;
P2 = P/2;
R1(count) = alpha*log2(1 + P1*G1/alpha);
R2(count) = (1-alpha)*log2(1 + P2*G2/(1-alpha));
count = count + 1;
end
hold on;
plot(R1,R2,'k');
xlabel('Rate of user 1 (bps/Hz)');
ylabel('Rate of user 2 (bps/Hz)');
grid on;
box on;
legend('NOMA','OFDMA')
```

**MATLAB code for Figure 8.**

```

clear all;

clc;

B = 5*10^6; %bandwidth Hz
N0 = 10^-21; %-150 dBw/Hz
N = N0*B; % dBW
G1 = 10^-12; %-120 dB
G2 = 10^-14; %-140 dB
Pcircuit = 100; %watt
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% NOMA
count = 1;
for p = 1:1:100 %W
P1 = p*0.1; %allocate less power to UE1
P2 = p - P1;
R1 = B*log2(1 + P1*G1/N);
R2 = B*log2(1 + P2*G2/(P1*G2 + N));
R = R1 + R2;
SE(count) = R/B; % bit/sec/Hz
EE(count) = (R/(Pcircuit + p)); % bit/watt.sec
count = count + 1;
end
hold on;
plot(SE,EE,'k');
xlabel('SE (bit/sec/Hz)');
ylabel('EE (bit/joule)');
grid on;
% OFDMA
count = 1;

```

```
greenpoint = 0;
maxEE = -1000;
for p = 1:1:100 %Watt
P1 = p/2;
P2 = p/2;
R1 = (B/2)*log2(1 + P1*G1/(N0*B/2));
R2 = (B/2)*log2(1 + P2*G2/(N0*B/2));
R = R1 + R2;
SE_line(count) = R/B; % bit/sec/Hz
EE_line(count) = (R/(Pcircuit + p)); % bit/watt.sec = Mbit/joule
count = count + 1;
end
hold on;
plot(SE_line,EE_line,'g-');
xlabel('SE (bit/sec/Hz)');
ylabel('EE (bit/joule)');
grid on;
```

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